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A. V. Lyulintsev (Saint Petersburg State University, Saint Petersburg, Russia)**. On some martingale constructions for PSI-processes.** [1]

Let $\xi_0, \xi_1, \dots$ be a sequence of random variables (r.v.'s), $\Pi(t)$ be a standard Poisson process independent of these r.v.'s, and $\lambda > 0$ be a fixed constant (intensity). A PSI-process (a Poisson stochastic index process) is defined as $\psi_\lambda(t) = \xi_{\Pi(\lambda t)}$ (see [1]).

We study joint martingale properties of the PSI–process $\psi_\lambda(t)$ and the integrated PSI-process $\Psi_\lambda(t) = \int_0^t \psi_\lambda(s)\, ds$ (see, e.g., [2]).

THEOREM. *Let $\xi_0, \xi_1, \dots$ be a sequence of independent and identically distributed (i.i.d.) r.v.'s, $\mathbf{E}\xi_0 = 0$, $\lambda > 0$. Let a filtration $\mathbf{F}$ be naturally generated by a Markov pair $(\psi_\lambda, \Psi_\lambda)$, i.e., $\mathcal{F}_t = \sigma\{(\psi_\lambda(s), \Psi_\lambda(s)),\, s \leqslant t\}$. Then the process $\lambda\Psi_\lambda(t) + \psi_\lambda(t)$, as $t \geqslant 0$, is a martingale relative to $\mathbf{F}$. Moreover,*

$$(1)\qquad \mathbf{E}\{\lambda\Psi_\lambda(t) + \psi_\lambda(t) \mid \psi_\lambda(s),\, \Psi_\lambda(s)\} = \lambda\Psi_\lambda(s) + \psi_\lambda(s), \qquad s \leqslant t,$$

since $(\psi_\lambda, \Psi_\lambda)$ is a Markov pair.

As a corollary, the case of stochastic intensity $\lambda > 0$ a.s. is considered.

In the case $\mathbf{D}\xi_0 < \infty$, the principal numerical characteristics of the martingale $\lambda\Psi_\lambda(t) + \psi_\lambda(t)$ are evaluated.

The trajectories of this martingale are modeled, in particular, for the cases of normal and Rademacher distribution ξ_0.

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A. P. Makarova (S. M. Kirov Military Medical Academy, St. Petersburg, Russia)**, V. A. Gorlov** (Voronezh, Russia)**, A. V. Makarova** (Air Force Academy named after N. E. Zhukovsky and Yu. A. Gagarin, Voronezh, Russia)**. Probabilistic modeling of network clusterization.**

An improved self-defined network (SDN) can be realized with the help of available algorithms and protocols [1]. In order to find the most efficient partition of the network, consider a partition $M \in \Phi$ of n vertices into m clusters (Φ is the set of all

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possible partitions of the vertex set of a graph V). We define some r.v. Q assuming values from 1 to m with probabilities q_i, $i = 1, \dots, m$. For each cluster i of the network, we define some r.v. P^i assuming values from 1 to n_i with probabilities p_i^k, where $k = 1, \dots, n_i$. As a result of the study, we introduce the expanded partition quality indicator $L(M)$ defined as the supremum of lengths of code words defining the partition quality M:

$$L(M) = \sum_{i=1}^{m} q_i \ln\Biggl(\sum_{i=1}^{m} q_i\Biggr) - 2\sum_{i=1}^{m} q_i \ln q_i - \sum_{\alpha=1}^{n} p_\alpha \ln p_\alpha + \sum_{i=1}^{m}\Biggl(q_i + \sum_{\alpha\in i} p_\alpha\Biggr) \ln\Biggl(q_i + \sum_{\alpha\in i} p_\alpha\Biggr).$$

THEOREM. *With this definition $L(M)$ in the algorithm, the malfunction probability of the network decreases, the performance indicator increases, and the "network life time" increases in general.*

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G. V. Martynov (Institute for Information Transmission Problems of the Russian Academy of Sciences (Kharkevich Institute), Moscow, Russia)**. The Cramér–von Mises test for parametric families of distributions.**

We consider the problem of testing the hypothesis that the distribution function of an observed r.v. belongs to a parametric family of distributions. To this end, the Cramér–von Mises test, the Kolmogorov–Smirnov test, and other tests can be applied. It is known that the limit distributions of these statistics do not depend on the unknown parameters of the observed r.v.'s for families of distributions of types $\mathcal{F} = \{F((x-m)/s)\}$ and $\mathcal{R} = \{R((x/\theta)^\kappa)\}$ (see [1], [2], [3]). Here, we consider the family $\mathcal{G} = \{G(x/\theta, \kappa),\, \theta, \kappa > 0\}$.

THEOREM. *Under certain regularity conditions on the family $\mathcal{G}$, the limit distribution of the Cramér–von Mises statistic depends on at most one parameter κ of the family.*

In particular, for the families $\mathcal{R}$ (see [3]), the distribution of the statistic does not depend at all on the parameters, and, for the family of gamma distributions, it depends only on the parameter κ.

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M. V. Melikian (Lomonosov Moscow State University, Russia). **Large system of oscillators with ultralocal action of a random stationary field.**

We consider a finite system of point particles of unit mass on the real line $\mathbf{R}$, where a particle with a fixed number n is subject to an external force $f(t)$, which is a stationary in a wide sense centered random process with continuous covariance function $B(s)$ and spectral measure $\mu(dx)$ (as in [1]).

THEOREM 1. *Let a measure μ be such that the covariance function of the random process under consideration can be written as $B(t)=\int_{\mathbf{R}} e^{itx}b(x)\,dx$. Then the mean energy of the entire system is bounded in time if at least one of the following conditions is met*:

(1) *The support of the function $b(x)$ is disjoint from the set $\{\nu_k,\ k=1,\dots,N\}$*;

(2) $(u_j,e_n)^2=0$ *for all j such that $\nu_j\in\operatorname{supp}b(x)$*;

(3) *if there is a single eigenvalue ν_j lying in $\operatorname{supp}b(x)$ and such that $(u_j,e_n)^2\neq 0$, then $\nu_j=0$ and $b(0)=b'(0)=0$.*

Otherwise, the mean energy grows in time, and there exists a positive constant C such that $\mathbf{E}(H(t))\sim Ct^2$.

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V. V. Misyura (Don State Technical University, Rostov-on-Don, Russia), **E. V. Misyura** (Plekhanov Russian State University of Economics, Moscow, Russia). **An application of order statistics for construction of one-step forecasting of times series.**

According to [1], the left and right boundaries of a one-step interval forecasting of the times series h_k, $k=1,\dots,N$, are formed, respectively, by the first k smallest order statistics and the statistics of order of $k+1$ to N obtained by shifting with window size τ, $1<\tau<N$, and with subsequent ordering. A quantile regression is proposed for determination of weights, because the interval estimate (h_l,h_k), $i\leqslant l<k\leqslant i+\tau-1$, for the median $\operatorname{Me}(h)_i^{i+\tau-1}$ of the random sequence $(h)_i^{i+\tau-1}=\{(h)_i,h_{i+1},\dots,h_{i+\tau-1}\}$, $i\leqslant l<k\leqslant i+\tau-1$, which specifies a symmetric interval with confidence level $1-2\alpha$ with $k=\tau-l-1+i$, is used for target variables. The following theorem justifies this choice of target variables.

THEOREM. *Let $\{H_{(1)},H_{(2)},\dots,H_{(\tau)}\}$ be order statistics for a sample $\{H_1,H_2,\dots,H_\tau\}$, and let r and s be such that $\mathbf{P}(H_{(r)}<h_p<H_{(s)})=1-2\alpha$ is a given confidence probability and the interval $(H_{(r)},H_{(s)})$ contains the unknown quantile $h_p=F^{-1}(p)$, $0<p<1$. Then the probability $\mathbf{P}(H_{(r)}<h_p<H_{(s)})$ does not depend on the unknown distribution function of the observed r.v. $F_H(h)$.*

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A. V. Nikitina, V. B. Dolgov (Don State Technical University, Rostov-on-Don, Russia). **Allelopathic interaction of hydrobionts based on a stochastic approach.** [2]

Constructive methods are proposed for compensation of the a priori uncertainty arising from the nonstationary and stochastic nature of ecological systems. The difference scheme for the homogeneous equations of the mathematical model of biological kinetics of a shallow water body (in the case of the Sea of Azov used as an example) with due account of the allelopathic interaction of aquatic organisms has the form [1]

$$\frac{C^{n+1} - C^n}{\tau} + A_x C^n + A_y C^n + A_z C^{n+\sigma} = 0,$$

where C is the impurity concentration; τ is the time increment; n is the time layer number; σ is the layout weight, $\sigma \in [0,1]$; and A_x, A_y, A_z are discrete analogues of the transfer operators along the coordinate axes Ox, Oy, Oz:

$$\begin{aligned}(A_x C)_i = u_{i+1/2}\frac{C_{i+1} - C_i}{2h_x} + u_{i-1/2}\frac{C_i - C_{i-1}}{2h_x} \\ - \mu_{i+1/2}\frac{C_{i+1} - C_i}{h_x^2} + \mu_{i-1/2}\frac{C_i - C_{i-1}}{h_x^2}, \qquad 0 \leqslant i \leqslant N,\end{aligned}$$

where h_x is the step of spatial variable, N is the number of steps, μ is the diffusion coefficient, and u is the water flow velocity. The discrete operators A_y, A_z have similar representations.

THEOREM. *Under the condition $\tau \leqslant \left(\max(2\mu/h_x^2 + 2\mu/h_y^2)\right)^{-1}$, the difference scheme is conditionally stable, and $\|C^{n+1}\| \leqslant \|C^0\|$.*

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A. K. Nikolaev (St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, St. Petersburg, Russia). **On the probabilistic representation of the resolvent of the two-dimensional Laplace operator.**

We consider the family of random linear operators

$$(1) \qquad \mathcal{R}_\lambda^t f(x) = \int_0^t e^{\lambda\tau}\left[\frac{1}{2\pi}\int_{S^1} f(x - \|w(\tau)\| \cdot \theta)\, dS(\theta)\right] d\tau, \qquad \operatorname{Re}\lambda \leqslant 0,$$

where $w(\tau)$, $\tau \geqslant 0$, $w(0) = (0,0)$, is a two-dimensional Wiener process.

The corresponding operator family arises in construction of a probabilistic representation of the resolvent of the two-dimensional Laplacian. We show that, with probability 1, the operators of this family are integral operators in $L_2(\mathbf{R}^2)$. We also study the kernels of these operators and construct an analogous operator family for the case $\operatorname{Re}\lambda \geqslant 0$.

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THEOREM. (1) *If* $\lambda \in \mathbf{C}$, $\operatorname{Re}\lambda < 0$, *then*

$$\left(-\frac{1}{2}\Delta - \lambda I\right)^{-1} f(x) = \mathbf{E}[\mathcal{R}_\lambda^\infty f(x)]$$

for all $f(x) \in L_2(\mathbf{R}^2)$.

(2) *If* $\lambda \in \mathbf{C} \setminus \sigma(\Delta/2)$, $\operatorname{Re}\lambda \geqslant 0$, *then*

$$\left(-\frac{1}{2}\Delta - \lambda I\right)^{-1} f(x) = (L_2) \lim_{t\to\infty} \mathbf{E}[\mathcal{R}_\lambda^t f(x)]$$

for all $f(x) \in L_2(\mathbf{R}^2)$.

(3) *If* $\lambda \in \sigma(-\Delta/2)$, *then the equality in* (2) *holds for all* $f(x) \in \mathcal{D}(-\Delta/2 - \lambda I)^{-1}$.

E. A. Pchelintsev, S. S. Perelevsky (Tomsk State University, Tomsk, Russia)**. On estimation for the trend coefficient of a diffusion process from discrete time observations.** [3]

On a probability space $(\Omega, \mathcal{F}, \mathbf{P})$, consider the stochastic differential equation $dy_t = S(y_t)\,dt + \sigma(y_t)\,dw_t$, $0 \leqslant t \leqslant T$, where $(w_t)_{t\geqslant 0}$ is a Wiener process, the initial value y_0 is a given constant, $\sigma(\,\cdot\,)$ is an unknown diffusion coefficient (the nuisance parameter), and $S(\,\cdot\,)$ is an unknown function from the class Σ defined in [1]. The problem is to estimate the trend $S(x)$, $x \in [a, b]$, from discrete observations $(y_{t_j})_{0\leqslant j\leqslant N}$, $t_j = j\delta$, with frequency $\delta = \delta_T$ and sample size $N = N_T$. A model selection procedure S^* is proposed based on improved weighted least squares estimates $(S^*_\lambda)_{\lambda\in\Lambda}$ introduced in [2]. These estimates are shown as having a better accuracy than those produced by the least squares method. The following theorem is proved.

THEOREM 1. *The mean-square risk of the model selection procedure* S^* *satisfies the nonasymptotic sharp oracle inequality*

$$\mathbf{E}_S\|S^* - S\|^2 \leqslant A(\rho_T) \min_{\lambda\in\Lambda} \mathbf{E}_S\|S^*_\lambda - S\|^2 + \frac{B_T}{\delta_T \rho_T},$$

where $\|\cdot\|$ *is the norm in* $L_2[a, b]$, $A(\rho_T) \to 1$, *and* $T^{-\epsilon} B_T \to 0$ *for any* $\epsilon > 0$ *as* $T \to \infty$.

Using Theorem 1 and the lower bound for the risk from [1], it is shown that the above estimate S^* is asymptotically efficient.

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A. Yu. Perevaryukha (St. Petersburg Federal Research Center of the Russian Academy of Sciences,, St. Petersburg, Russia). **Invasion modeling with stochastically perturbed delay.** [4]

In rapid invasions and infections it may happen that the reached population size $N(t) \to \mathcal{K}$ is not stable. The stochastic perturbation of dynamics becomes significant if the confrontation is activated in a state critical for the environment. When approaching the threshold of destruction of the environment, an increase in counteraction is observed, which is typical of the body's immune response. A counteraction is known to increase as the environmental destruction threshold is approached, which is a typical response of the human immune system. The activation time is important and variable but does not exceed τ_1. Assume that τ_1 is varied by an r.v. γ in a bounded region. We propose the following invasion model with delay $(t - \tau_1\gamma)$ perturbed by a uniform r.v.:

$$(1)\quad \frac{dN}{dt} = rN(t)\ln\left(\frac{\mathcal{K}}{N(t-\tau\gamma)}\right) - \frac{\delta N^2(t-\tau_1\gamma)}{(J-N(t))^2} - qN(t), \qquad \delta > q, \quad \gamma(\omega) \in [1,2].$$

A sharp transition to a deep population crisis $N(t) \to 0 + \epsilon$ takes place as $N(t)$ approaches the threshold J, $N(0) < J < \mathcal{K}$. A scenario for overcoming the crisis with formation of oscillations $N(t) \to N_*(t)$, $\max N_*(t) < J$, depends on stochastic time factors. According to (1), the population is guaranteed to become extinct at the local area with increased reproduction potential r.

Theorem. *There exists* $r = \overline{r}$ *such that the event* $\lim_{t\to\overline{t}} N(t; \overline{r}\,\tau) = 0$ *has positive probability, and there exist* $\widehat{r} > \overline{r}$ *and* $t < \infty$ *such that the above event is realized with probability* 1 ($\widehat{r}$ *is the critical threshold of reproductive activity*). *Model* (1) *describes scenarios in which the immune system struggles with an infection, which may become chronic for* $N(t) \ll J$. *The immune response is not completely predetermined due to nondeterministic duration of the immune activation stages. The antigen presentation times and the duration of selection of suitable "naive" lymphocyte cells are varied.*

G. A. Popov (Lomonosov Moscow State University, Moscow, Russia). **Limit distributions of random walks on multidimensional lattices.** [5]

We prove a limit theorem, as $t \to \infty$, for the size μ_t of population of particles in a critical branching random walk (BRW) on $\mathbf{Z}$ with transition probabilities $p(t,x,y) \sim h_{1,\alpha}\, t^{-1/\alpha}$, where $h_{1,\alpha} > 0$ and $\alpha \in [1,2)$. The survival probability for the population of this BRW was studied, e.g., in [1].

Theorem. *For a critical recurrent BRW, for any* $z > 0$ *and* $x \in \mathbf{Z}$,

$$\lim_{t\to\infty} \mathbf{E}_x[e^{-z\mu_t} \mid \mu_t > 0] = 1 - \sqrt{1 - e^{-z}}.$$

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E. O. Rahimbaeva, A. M. Atayan (Don State Technical University, Rostov-on-Don, Russia). **Processing of noisy images and data based on recursive filtration.** [6]

Consider the mathematical model of biogeochemical cycles of a shallow water body [1] having the form

$$\frac{\partial q_i}{\partial t} + u\,\frac{\partial q_i}{\partial x} + v\,\frac{\partial q_i}{\partial y} + w\,\frac{\partial q_i}{\partial z} = \operatorname{div}(k\,\operatorname{grad} q_i) + R_{q_i},$$

where q_i is the assembly of the ith component [mg/l]; $i \in M$, $M = \{\mathrm{F}_1, \mathrm{F}_2, \mathrm{F}_3, \mathrm{PO}_4, \mathrm{POP}, \mathrm{DOP}, \mathrm{NO}_3, \mathrm{NO}_2, \mathrm{NH}_4, \mathrm{Si}\}$; $\{u, v, w\}$ are the components of the flow velocity vector H_2O [m/s]; k is the turbulent exchange coefficient [$\mathrm{m}^2\cdot\mathrm{s}$]; and R_{q_i} is the source function of biogenic components [mg/l·s)]. This equation is augmented with corresponding initial and boundary conditions.

The optimal solution to the problem of variational assimilation of data for the mathematical model of biogeochemical cycles is shown to be stable based on a study of the sensitivity coefficients as norms of the response operators.

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N. Ratanov (Chelyabinsk State University, Chelyabinsk, Russia). **On Kac–Ornstein–Uhlenbeck processes.** [7]

A connection is established between Markov-modulated Lévy processes, existence of invariant distributions, and exponential functionals. Necessary and sufficient conditions for existence of invariant distributions for such processes are obtained. We also prove the following result that illustrates [1].

Theorem. *Let $\varepsilon(t)$ be a two-state Markov process. The stationary distribution (if it exists) of the process $\langle Z, \varepsilon\rangle$, as given by the equation $Z(t) = z + \int_0^t \bigl(a_{\varepsilon(u)} - c_{\varepsilon(u)} Z(u)\bigr)\,\mathrm{d}u$, is uniquely defined by the distribution of the exponential functional (see* [2]*)*

$$G = \int_0^\infty \exp\biggl(-\int_0^t c_{\varepsilon(u)}\,\mathrm{d}u\biggr) a_{\varepsilon(t)}\,\mathrm{d}t.$$

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D. B. Rokhlin (Southern Federal University, Rostov-on-Don, Russia)**. On incentive pricing algorithms under the lack of information about agent utilities.** [8]

We consider a leader, who prices a resource or goods and tries to change the behavior of selfish agents in a desired way. In particular, we consider a corporation producing and selling d commodities and consisting of n production and m sales divisions. Let $F(x,y)$ be the income of the corporation depending on vectors $x, y \in \mathbf{R}^d$, which describe the amounts of commodities to be produced and sold, and let $(\widetilde{x}(\lambda), \widetilde{y}(\lambda))$ be division reactions to transfer prices within the corporation. The following recurrence formula was obtained by applying the algorithm from [1] to the dual problem of income maximization:

$$\lambda_t = -\frac{\sum_{j=1}^{t-1} \Delta\widetilde{z}(\lambda_j)}{\sqrt{\sum_{j=1}^{t-1} \|\Delta\widetilde{z}(\lambda_j)\|^2}}, \quad \lambda_0 = 0; \qquad \Delta\widetilde{z}(\lambda) := \sum_{i=1}^{n} \widetilde{y}_i(\lambda) - \sum_{i=1}^{m} \widetilde{x}_i(\lambda).$$

Theorem. *Let F^* be the maximum income. Then the average transfer price vector $\overline{\lambda}_T = (1/T)\sum_{t=1}^{T} \lambda_t$ satisfies*

$$F^* - F(\widetilde{z}(\overline{\lambda}_T)) \leqslant CT^{-1/4}, \quad \|\Delta\widetilde{z}(\overline{\lambda}_T)\| \leqslant CT^{-1/4}.$$

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O. V. Rusakov (Saint Petersburg State University, Saint Petersburg, Russia)**, Yu. V. Yakubovich** (Saint Petersburg State University, Saint Petersburg, Russia)**. Ambit, trawl, and PSI processes.** [9]

By a PSI (Poisson stochastic index) process, here we mean the following subordinator: $\psi(t) = \xi_{\Pi(\lambda t)}$, $t \geqslant 0$, where $(\xi) = \xi_0, \xi_1, \ldots$ is a sequence of i.i.d. r.v.'s, and Π is a standard Poisson process independent of (ξ), $\lambda > 0$. Assume that ξ_0 belongs to the attraction domain of some symmetric α-stable law, $\alpha \in (0,2]$. Consider the independent copies of the PSI processes (ψ_j), $j \in \mathbf{N}$.

Theorem. *The following convergence of finite-dimensional distributions holds*:

$$\frac{1}{N^{1/\alpha}} \sum_{j=1}^{N} \psi_j(t) \Rightarrow U^{\alpha}(t), \qquad N \to \infty, \quad t \geqslant 0. \tag{1}$$

Here $U^{\alpha}(t) = \int_{A_\eta(t)} \mathrm{d}\mathcal{L}_\alpha(u,v)$, where $\mathcal{L}_\alpha$ is a Lévy basis (*see* [1]) *on $\mathbf{R}_+ \times \mathbf{R}$ with symmetric α-stable law and structural Lebesgue measure. The so-called ambit sets* (*see* [1]) *are defined by $A_\eta(t) = \{(x,s)\colon s \leqslant t,\ 0 \leqslant x \leqslant \eta(s-t)\}$, $t \in \mathbf{R}$, and, in our case, i.e., for the limits of PSI processes, $\eta(-r) = \lambda\exp(-\lambda r)$, $r \in \mathbf{R}_+$.*

The limit process U^α is stationary, and it can be considered on $\mathbf{R} \ni t$. In the notation of [1], the random process U^α is called a trawl process with monotonic trawl, that is, the sets $\{A_\eta(t)\}$, $t \in \mathbf{R}$.

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V. V. Rykov (Gubkin Russian State University of Oil and Gas (National Research University), Moscow, Russia; Peoples' Friendship University of Russia, Moscow, Russia). **On decomposable semiregenerative processes and their application to double redundant renewable systems.** [10]

The following theorem is based on [1], [2].

Theorem. *The Laplace transform $\widetilde{R}(s)$ of the reliability function $R(t)$ of a redundant system with arbitrarily distributed lifetime $A(t)$ and recovery time $B(t)$ of its units has the form*

$$\widetilde{R}(s) = \frac{(1-\widetilde{a}(s))(1+\widetilde{a}(s)-\widetilde{a}_B(s))}{s(1-\widetilde{a}_B(s))},$$

where the modified Laplace–Stieltjes transforms of the distributions $A(t)$ and $B(t)$,

$$\widetilde{a}_B(s) = \int_0^\infty e^{-sx} B(x)\, dA(x), \qquad \widetilde{b}_A(s) = \int_0^\infty e^{-sx} A(x)\, dB(x),$$

are introduced in parallel with the standard transforms $\widetilde{a}(s) = \int_0^\infty e^{-sx}\, dA(x)$, $\widetilde{b}(s) = \int_0^\infty e^{-sx}\, dB(x)$.

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D. S. Sergeeva (Voronezh State University, Voronezh, Russia). **On global in time solutions of one class of differential–algebraic equations with random perturbations.**

The following theorem extends some results of [1] and [2]. Let $\widetilde{L}$ and $\widetilde{M}$ be, respectively, generated and nongenerated matrices. Consider the equations

$$\begin{cases} \widetilde{L} D_S \xi(t) = \widetilde{M}\xi(t) + \widetilde{f}(t,\xi(t)), \\ D_2\xi(t) = \overline{\Theta}(\xi), \end{cases} \tag{1}$$

$$\begin{cases} D_S \eta^{(1)}(t) = J\eta^{(1)}(t) + f^{(1)}(t,\eta(t)), \\ D_2\eta^{(1)}(t) = \Xi. \end{cases} \tag{2}$$

Theorem. *A necessary and sufficient condition that the forward and backward flows generated by (1) be simultaneously complete and continuous at infinity is that there exist positive smooth proper functions $u(t,x)$ and $\overline{u}(t,x)$ on $[0,\infty)\times \mathbf{R}^d$ such that $(\partial/\partial t + \mathcal{A})u < C$ and $(-\partial/\partial t + \overline{\mathcal{A}})\tilde{u} < \overline{C}$ for all (t,x) for some positive constants C and $\overline{C}$, where $\mathcal{A}$ and $\overline{\mathcal{A}}$ are generators of the forward and backward flows generated by (2).*

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M. M. Shumafov (Adyghe State University, Maikop, Russia)**, V. B. Tlyachev, T. A. Panesh, and M. A. Havaja. On stability in probability of some second-older stochastic differential equations.**

Our theorems extend and supplement some results of [1], [2]. We give one of these theorems.

Theorem. *Suppose that there exist numbers* $b > 0$ *and* $c \in \mathbf{R}$ *such that, for functions* $f(y)$, $g(x)$, *and* $\sigma(x)$, $x \in \mathbf{R}$, *the following conditions are satisfied*:

(1) f, g, σ *satisfy the Lipschitz condition on* $\mathbf{R}$;
(2) $f(y)/y > b$ *for all* $y \neq 0$, $f(0) = 0$;
(3) $xg(x) > 0$ *for all* $x \neq 0$, $g(0) = 0$;
(4) $\int_0^x g(s)\,ds \to +\infty$ *as* $|x| \to \infty$;
(5) $0 < \sigma(y)/y < c^2$ *for all* $y \neq 0$ *and* $c^2 < 2b$, $\sigma(0) = 0$.

Then the trivial solution $(x(t) \equiv 0,\ y(t) \equiv 0)$ *of the stochastic Itô system*

$$dx(t) = y(t)\,dt, \qquad dy(t) = [-f(y) - g(x)]\,dt + \sigma(y)\,dw(t),$$

where $w(t)$ *is a one-dimensional Wiener process, is asymptotically stable in probability in the large.*

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A. R. Simonyan (Sochi State University, Sochi, Russia)**, E. I. Ulitina** (Sochi State University, Sochi, Russia)**. Virtual standby times in the Kleinrock model.**

Consider the Kleinrock model [1], [2] with arrival rate $a_0, \dots, a_r > 0$ and the distribution function of customer service $B_k(x)$, $B_k(+0) = 0$, $k = 1, \dots, r$. Let $w_k(t)$ be the virtual standby time of the kth customer at time t [1].

In the present note, we propose a new method for analysis of $w_k(t)$, $k = \overline{1, r}$, $t \geqslant 0$.

For $k = 1, \dots, r$, $s \geqslant 0$, $t \geqslant 0$, the following equalities hold [2]:

$$\omega_k(s,t) = \overline{\omega}_k(m_{k-1}(s), t).$$

We set $p_k(s) = s - \sum_{i=1}^k a_i(1-\beta_i(s))$, $p_k^j(s) = s - \sum_{i=1}^k a_{ij}(1-\beta_i(s))$, $k = 1, \dots, r$, $j = k, \dots, r$, $s \geqslant 0$. The following theorem holds.

THEOREM. *For any* $k = 1, \dots, r$, $t \geqslant 0$, $s \geqslant 0$,

$$\overline{\omega}_k(s,t) = e^{p_k(s)t}\Big\{1 - s\int_0^t e^{-p_k(s)u}P(u)\,du - \sum_{j=k+1}^{r} a_j(1-\beta_j(s))\int_0^t e^{-p_k(s)v}\,dv\int_0^{(b_k/(b_k-b_j))(t-v)} e^{-p_k^{j-1}(s)u}\,d_u\mathbf{P}(w_j(v)<u)\Big\},$$

where $\int_0^\infty e^{-st}P(t)\,dt = (m_r(s))^{-1}$, $s \geqslant 0$ *(*b_k, $m_k(s)$, $\beta_k(s)$ *are defined in* [1]*).*

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N. V. Smorodina (St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, St. Petersburg, Russia)**. On kernels of random operators.** [11]

Let $\xi_x(t)$ be the solution of the stochastic differential equation

$$d\xi_x(t) = b(\xi_x(t))b'(\xi_x(t))\,dt + b(\xi_x(t))\,dw(t), \quad \xi_x(0) = x.$$

In the space $L_2(\mathbf{R})$, consider the operator $\mathcal{A} = -(1/2)(d/dx)\big(b^2(x)d/dx\big) + V(x)$ defined on the domain $W_2^2(\mathbf{R})$. We assume that the functions $b(x)$, $V(x)$ satisfy the following conditions:

(1) $V \in L_1(\mathbf{R})$;
(2) $b \in C_{\mathrm{b}^2}$ and is separated from zero;
(3) there exists $b_0 > 0$ such that $\lim_{x\to\pm\infty} b(x) = b_0$;
(4) $\lim_{x\to\pm\infty} b'(x) = \lim_{x\to\pm\infty} b''(x) = 0$;
(5) $\int_{\mathbf{R}} x^2(|b(x) - b_0| + |b'(x)|)\,dx < \infty$.

From (1)–(5) it follows that the spectrum of the operator $\mathcal{A}$ consists of the interval $[0,\infty)$ and, possibly, several negative single eigenvalues. By P_{a} we denote the orthogonal projection onto the absolutely continuous subspace H_{a} of the operator $\mathcal{A}$. We also set $\mathcal{A}_0 = \mathcal{A}P_{\mathrm{a}}$.

For each λ satisfying $\operatorname{Re}\lambda \leqslant 0$ we define the random operator $\mathcal{R}_\lambda^t$ by the formula $\mathcal{R}_\lambda^t f(x) = \int_0^t e^{\lambda\tau}(P_a f)(\xi_x(\tau))\exp\big\{-\int_0^\tau V(\xi_x(s))\,ds\big\}\,d\tau$.

THEOREM 1. (1) *With probability* 1, *the operator* $\mathcal{R}_\lambda^t$ *is bounded by the integral operator* $\mathcal{R}_\lambda^t f(x) = \int_{\mathbf{R}} r_\lambda(t,x,y)f(y)\,dy$ *in* $L_2(\mathbf{R})$; *moreover, for* $\operatorname{Re}\lambda < 0$, *the last equality also holds for* $t = \infty$.

(2) *For all* λ, t, x, *the function* $r_\lambda(t,x,\cdot\,)$ *belongs to* W_2^α *for any* $\alpha \in [0,1/2)$.

THEOREM 2. (1) *If* $\operatorname{Re}\lambda < 0$, *then for any* $f \in H_{\mathrm{a}}$,

$$\mathbf{E}\int_{\mathbf{R}} r_\lambda(\infty,\cdot\,,y)f(y)\,dy = (\mathcal{A}_0 - \lambda I)^{-1}f.$$

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(2) *If* $\operatorname{Re}\lambda = 0$ *and* $\lambda \neq 0$, *then, for any* $f \in H_{\mathrm{a}}$,

$$\lim_{t\to\infty} \mathbf{E} \int_{\mathbf{R}} r_\lambda(t,\cdot,y) f(y)\,dy = (\mathcal{A}_0 - \lambda I)^{-1} f.$$

V. N. Sobolev, A. E. Kondratenko (Lomonosov Moscow State University, Moscow, Russia)**. Generalization of one Senatov's result in the central limit theorem.**

The following theorem extends Theorems 4 and 5 of [1].

THEOREM. *Let* $\xi_1, \xi_2, \dots$ *be symmetric i.i.d. r.v.'s with* $\mathbf{E}\xi = 0$, $\mathbf{D}\xi = 1$, *finite moments of even orders* $m+2 \geqslant 2$, *and a characteristic function* $f(t)$ *such that* $|f(t)|^\nu$ *is integrable on* $\mathbf{R}$ *for some* $\nu > 0$. *Then, for the density* $p_n(x)$ *of the normalized sums* $(\xi_1 + \dots + \xi_n)n^{-1/2}$, *for* $n \geqslant \max\{m, \nu\}$ *and* $x \in \mathbf{R}$,

$$\biggl|p_n(x) - \varphi(x) \sum_{s=0}^{m/2} C_n^s \sum_{l=4s}^{m-4+4s} \frac{\Theta_{s,l}}{n^{l/2}} H_l(x) - \frac{\theta_{m+2}^{(\lambda)}}{n^{m/2}}\,\varphi(x) H_l(x)\biggr| \lesssim \frac{\overline{\lambda}}{(m+2)!}\,\frac{\mathbf{E}\xi^{m+2}}{n^{m/2}}\,\frac{B_{m+2}}{\sqrt{2\pi}},$$

where $\varphi(x)$ *is the density,* B_{m+2} *is the moment of order* $m+2$ *of the standard normal distribution,* $H_l(x) = (-1)^l \varphi^{(l)}(x)/\varphi(x)$ *is the Chebyshev–Hermite polynomial of degree* l, $0 \leqslant \lambda < 1$, $\overline{\lambda} = \max\{\lambda,\, 1-\lambda\}$, *and, for* $k_j \geqslant 4$,

$$\theta_k = \sum_{j=0}^{[k/2]} \frac{(-1)^j}{2^j j!}\,\frac{\mathbf{E}\xi^{k-2j}}{(k-2j)!},\quad \theta_{m+2}^{(\lambda)} = \theta_{m+2} - \frac{(1-\lambda)\,\mathbf{E}\xi^{m+2}}{(m+2)!},\quad \Theta_{s,l} = \sum_{k_1+\dots+k_s=l} \theta_{k_1}\cdots\theta_{k_s}.$$

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M. A. Stepovich, V. V. Kalmanovich (Tsiolkovsky Kaluga State University, Kaluga, Russia)**, D. V. Turtin** (Ivanovo State University, Ivanovo, Russia)**, E. V. Seregina** (Kaluga Branch of Bauman Moscow State Technical University (National Research University), Kaluga, Russia)**. On some results of mathematical modeling of diffusion processes due to interaction of charged particles and/or electromagnetic radiation with semiconductor structures.** [12]

Earlier [1], we considered stochastic diffusion models and subsequent radiative recombination of nonequilibrium minority carriers generated in homogeneous semiconductors by wide electronic or light beams [2]. In this note, for the mathematical models of diffusion and cathodoluminescence, we put forward estimates capable of assessing the correctness of these models and, from a random variation in the right-hand part of the differential diffusion equation, evaluate the changes in the solution of this equation and the changes in the cathodoluminescence parameters. Model

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calculations were carried out for target parameters characteristic of promising materials of semiconductor optoelectronics.

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D. A. Suchkova, F. S. Nasyrov (Ufa State Aviation Technical University, Ufa, Russia)**. On the Korteweg de Vries equation with noise in the variance and in the nonlinear term.**

Consider the Korteweg de Vries equation with noise in the form of a Stratonovich stochastic integral

$$d_t u + u u_x\, dt + u_{xxx}\, dt + \varepsilon u u_x * dW(t) + \varepsilon u_{xxx} * dW(t) = 0, \tag{1}$$

where $\varepsilon > 0$, $u = u(t, W(t), x)$, $u(0,0,x) = u_0$, $(x,t) \in \mathbf{R} \times [0,T]$, and $d_t u = u_t\, dt + u_v * dW(t)$.

Theorem 1. *Any solution of* (1) *is represented as* $u(t, W(t), x) = \varphi(t+\varepsilon W(t), x)$ (*see* [1]), *where* $W(t)$ *is a standard Wiener process.*

Corollary. *A particular solution of* (1) *in the form of a solitary wave* (*soliton*) *exists and is represented in the form* (*see* [2])

$$\varphi(t+\varepsilon W(t), x) = A \cosh^{-2}\left(\frac{x - (A/3)(t+\varepsilon W(t))}{\Delta}\right), \quad A = \text{const}, \quad \Delta = \sqrt{\frac{12}{A}}. \tag{2}$$

Theorem 2. *Any solution of* (1) *is represented as* $u(t, W(t), x) = \varphi(t+\varepsilon W(t), x)$, *where* $W(t)$ *is an arbitrary continuous with probability* 1 *random process or a continuous deterministic function.*

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A. I. Sukhinov (Don State Technical University, Rostov-on-Don, Russia)**, S. V. Protsenko** (A. P. Chekhov Taganrog State Institute (Taganrog Branch of Rostov State University of Economics, Taganrog, Russia)**. Construction of a turbulent mixing model for coastal systems based on statistical analysis of expedition data.** [13]

A tray water model of turbulent mixing [1] is constructed. The vertical turbulent exchange ratio is parameterized using experimental data obtained during the expedition using an ADCP Workhorse Sentine 600 ADCP. Readings are obtained from 17

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Sea Azov stations with 1 s scanning frequency (128 measurements of each of the three depth components of the velocity vector).

Experiments are performed for parameterization and determination of the turbulent exchange coefficient based on statistically processed data:

$$\nu = (0.41z)^2 \cdot 0.5 \sqrt{\left(\frac{\partial \overline{u}}{\partial z}\right)^2 + \left(\frac{\partial \overline{v}}{\partial z}\right)^2}, \qquad \nu = 0.5\,(C\Delta)^2 \sqrt{\left(\frac{\partial \overline{u}}{\partial z}\right)^2 + \left(\frac{\partial \overline{v}}{\partial z}\right)^2};$$

here, ν is the turbulent exchange coefficient in the vertical direction; $\overline{u}$, $\overline{v}$ are time-averaged pulsations of the velocity components of the water flow; C is an empirical constant; and Δ is the characteristic grid scale.

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A. N. Tikhomirov (Komi Scientific Center of Ural Branch of RAS, Syktyvkar, Russia). **Limit theorems for Laplace matrices and adjacency matrices of random graphs.**

Let $\mathbf{A}$ and $\mathbf{L}$ be, respectively, the adjacency and Laplace matrices of the weighted random graph $\{V, E, W\}$ with $|V| = n$, $\mathbf{E}W_{ij} = 0$, $\mathbf{E}W_{ij}^2 = \sigma_{ij}^2$, and $\mathbf{P}\{(i,j) \in E\} = p_{ij}^{(n)}$. Next, let $\lambda_1, \dots, \lambda_n$ and $\mu_1, \dots, \mu_n$ be the eigenvalues of the matrices $\mathbf{A}$ and $\mathbf{L}$, respectively. Consider the normalizing factor $a_n := (1/n)\sum_{j=1}^n \sum_{k=1}^n p_{jk}^{(n)} \sigma_{jk}^2$ and define the empirical spectral distribution functions of the adjacency and Laplace matrices $F_n(x) := (1/n)\sum_{j=1}^n I\{\lambda_j < x\sqrt{a_n}\}$, $G_n(x) = (1/n)\sum_{j=1}^n I\{\mu_j < x\sqrt{a_n}\}$, respectively, where $I\{A\}$ is the indicator of event A. The following theorem extends some results of [1].

Theorem. *Let* $\lim_{n\to\infty} a_n = \infty$, $\sup_{n\geqslant 1}(\max_{1\leqslant j,k\leqslant n} p_{jk}\sigma_{jk}^2/a_n) < \infty$, *and let the following conditions be met*: $\lim_{n\to\infty}(1/(na_n))\sum_{j=1}^n \sum_{k=1}^n |p_{jk}\sigma_{jk}^2 - a_n/n| = 0$,

$$\lim_{n\to\infty} \frac{1}{na_n} \sum_{j=1}^n \sum_{k=1}^n \mathbf{E}X_{jk}^2 I\{|X_{jk}| > \tau\sqrt{a_n}\} = 0.$$

Then $F_n(x)$ and $G_n(x)$ weakly converge in probability, respectively, to the semicircular law and to the free convolution of the distribution functions of the normal and semicircular laws.

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M. S. Tikhov (National Research Lobachevsky State University of Nizhny Novgorod, Russia)**. Quantile estimation of the distribution function using Bernstein polynomials.**

Let $X_1, X_2, \dots, X_n$ be latent i.i.d. r.v.'s with unknown continuous distribution function $F(x)$ and density function $f(x) > 0$, $x \in (0,1)$; let $[0,1]$ be the support of this distribution; let $u_i = i/n$, $i = 0, 1, \dots, n$, be the division points on $[0,1]$; and let $W_i = I(X_i < u_i)$ be the indicator of the event $\{X_i < u_i\}$. We consider the problem of estimation of a quantile of order $0 < \lambda < 1$ of the distribution function $F(x)$ from the sample $\mathcal{W}^{(n)} = \{(u_i, W_i),\ i = 0, \dots, n\}$. This problem comes from biology and is known as the "dose–effect" dependence.

For complete samples, estimates of the distribution function $F(x)$ were investigated in [1] using the Bernstein polynomials $b_k(n,x) = C_n^k x^k (1-x)^{n-k}$. In [2], the statistic $F_n^*(x) = \sum_{k=0}^n W_k b_k(n,x)$ was used as an estimator for $F(x)$ from a sample $\mathcal{W}^{(n)}$. For a given $0 < \lambda < 1$, we define $x_\lambda = \inf\{x\colon F(x) \geqslant \lambda\}$, $\widehat{x}_{n,\lambda} = \inf\{x\colon F_n^*(x) \geqslant \lambda\}$. The statistic $\widehat{x}_{n,\lambda}$ is considered as an estimate for the quantile x_λ of order $0 < \lambda < 1$ of the distribution function $F(x)$ in the "dose–effect" dependence. Let $\sigma^2 = \lambda(1-\lambda)/(4\pi f^2(x_\lambda) x_\lambda (1 - x_\lambda))$. The following result holds.

THEOREM. *Assume that* $F(x)$ *has bounded third derivative and* $0 < \lambda < 1$ *is given. Then*

$$\widehat{x}_{n,\lambda} \xrightarrow[n\to\infty]{p} x_\lambda, \qquad \sqrt{n}\,(x_{n,\lambda} - x_\lambda) \xrightarrow[n\to\infty]{d} N(0, \sigma^2).$$

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G. A. Vlaskov (Don State Technical University, Rostov-on-Don, Russia)**. On the algorithm of numerical simulation of electronic density of stochastically convecting polar ionosphere.**

In the continuity equation $\partial N_e/\partial t + (\vec{v} + \vec{v}_{\rm st})\nabla N_e = q - \beta N_e$ in [1], where N_e is the electron density, β is the recombination ratio, and q is the ion formation function, the field of transport velocities is split into the deterministic $\vec{v}$ and the stochastic $\vec{v}_{\rm st}$ components. The stochastic component is represented by the Wiener process $\sigma(x,t)W(t)$. In [1], an algorithm is given for evaluation of the electron density as a random function based on the Monte Carlo method.

In the present note, we justify the use of $\sigma(x,t)$. To this end, we analyze the data from [2]. The fluctuations $\vec{E}_{\rm st}$ can be as high as $25\,$mV/m in the auroral zone and $5\,$mV/m in the polar cap. This gives, respectively, $v_{\rm st} = 500\,$m/s and $v_{\rm st} = 100\,$m/s. With $10\,$km for the typical scale L_0, we get $\sigma = 2500$ for the auroral zone and $\sigma = 500$ for the polar cap.

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T. A. Volosatova, N. V. Neumerzhitskaia, I. V. Pavlov, S. I. Uglich (Don State Technical University, Rostov-on-Don, Russia). **Development of a model with random priorities.**

For a model with random priorities, the following theorem extends some results of [1], [2].

Theorem. *For parameters* $c_1 > 0, \dots, c_n > 0$, *consider the function*

$$F(u_1, \dots, u_{n-1}) = \mathbf{E}^{\mathbf{P}}\bigl(u_1^{\alpha_1} \cdots u_{n-1}^{\alpha_{n-1}} \cdot (-c_1 u_1 - \cdots - c_{n-1} u_{n-1} + c_n)^{\alpha_n}\bigr),$$

defined on the domain D *defined by* $u_1 > 0, \dots, u_{n-1} > 0$, $c_1 u_1 + \cdots + c_{n-1} u_{n-1} < c_n$ ($\alpha_1, \dots, \alpha_n$ *are r.v.'s on some probability space* $(\Omega, \mathcal{F}, \mathbf{P})$). *Assume that* $\alpha_i > 0$ *and* $\sum_{1 \leqslant k \leqslant n,\, k \neq i} \alpha_k < 1$ $\mathbf{P}$-*a.s. for any* i $(i = 1, \dots, n)$. *Then the function* F *is strictly concave on* D.

Corollary. *Under the conditions of the above theorem, the function* F *in the domain* D *has a unique local maximum which, simultaneously, is a global maximum.*

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A. L. Yakymiv (Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia). **Limit behavior of ordered statistics on the cycle lengths in random *A*-permutations.** [14]

We fix a set of natural numbers A. By $T_n(A)$ we denote the set of permutations of degree n whose cycle lengths lie in A (the so-called A-permutations). We consider a random permutation τ_n uniformly distributed on the set $T_n(A)$. Let ζ_n be the total number of cycles and $\eta_n(1) \leqslant \eta_n(2) \leqslant \cdots \leqslant \eta_n(\zeta_n)$ be the ordered statistics on the cycle lengths in the permutation τ_n. We assume that the sequence $|T_n(A)|/(n-1)!$ varies regularly at infinity with index $\varrho > 0$. We fix a real x and define, for $m \in \mathbf{N}$ and $t > 0$,

$$r = \exp\biggl(\frac{m}{\varrho} + x\frac{\sqrt{m}}{\varrho}\biggr), \qquad l(t) = \sum_{i \in A,\, i \leqslant t} \frac{1}{i}.$$

Theorem 1. *Let* $(\varrho \ln n - m)/\sqrt{\ln n} \to +\infty$ *as* $n \to \infty$. *Then*

$$\mathbf{P}\{\varrho \ln \eta_n(m) \leqslant m + x\sqrt{m}\,\} = \Phi(z) + \frac{1}{405 l^{3/2}(r)}\bigl(10\Phi^{(3)}(z) + \Phi^{(5)}(z)\bigr) + O\biggl(\frac{1}{\ln^2 n}\biggr);$$

$$z = y - \frac{1}{108\mu}(y^3 - y), \quad y = 3\sqrt{\mu}\,\Biggl(\biggl(\frac{\nu}{\mu}\biggr)^{1/3} - \frac{1}{9\mu} - 1\Biggr), \quad \mu = m + 1, \quad \nu = l(r).$$

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This theorem generalizes the main result of [1] to the case $A \neq \mathbf{N}$.

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E. B. Yarovaya (Lomonosov Moscow State University, Moscow, Russia)**. Limit behavior of particle populations in a branching random walk.** [15]

For a critical branching random walk (BRW) on $\mathbf{Z}^d$ from [2], [3] with transition probabilities $p(t,x,y)$, the theorem given below implies that the growth of the conditional expectations for the populations $\mu(t,x)$ and the subpopulations $\mu(t,x,y)$ of particles is slower than that with the presence of sources of the same intensity at each lattice point (see [1]).

Theorem. *For a critical recurrent BRW on $\mathbf{Z}^d$ with one branching source of particles with $\mu(0,x,y)=\delta_y(x)$ and $t\to\infty$, the following assertions hold*:

(a) *If $p(t,x,y)\sim\gamma_d t^{-d/2}$, $\gamma_d>0$, $d=1$ or $d=2$, then*

$$\mathbf{E}[\mu(t,x)\,|\,\mu(t,x)>0]\sim K_d(x)v_d(t),\quad \mathbf{E}[\mu(t,x,y)\,|\,\mu(t,x)>0]\sim K_d(x,y)v_d^*(t);$$

(b) *if $p(t,x,y)\sim h_{d,\alpha}t^{-d/\alpha}$, $h_{d,\alpha}>0$, $\alpha\in[1,2)$, $d=1$, then*

$$\mathbf{E}[\mu(t,x)\,|\,\mu(t,x)>0]\sim V_{d,\alpha}(x)u_{d,\alpha}(t),\quad \mathbf{E}[\mu(t,x,y)\,|\,\mu(t,x)>0]\sim V_{d,\alpha}(x,y)u_{d,\alpha}^*(t),$$

where $v_1(t)\sim t^{1/4}$, $v_1^*(t)\sim t^{-1/4}$, $v_2(t)=u_{1,1}(t)\sim\sqrt{\ln t}$, $v_2^*(t)=u_{1,1}^*(t)\sim\sqrt{\ln t}/t$, $u_{1,\alpha}\sim t^{(\alpha-1)/(2\alpha)}$, $u_{1,\alpha}^*\sim t^{(\alpha-3)/(2\alpha)}$, $\alpha\in(1,2)$.

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V. G. Zadorozhnii (Voronezh State University, Voronezh, Russia)**. Optimal control of a linear stochastic system.**

We consider a control linear system of differential equations $dx/dt=\varepsilon(t)Ax+bu(t)$, $x(t_0)=x_0$ with the efficiency criterion for the control

$$I=\frac{1}{2}\int_{t_0}^{t_1}\int_{t_0}^{t_1}\big[\langle B(s_1,s_2)\mathbf{E}[x(s_1)],\mathbf{E}[x(s_2)]\rangle+C(s_1,s_2)\mathbf{E}[u(s_1)]\mathbf{E}[u(s_2)]\big]\,ds_1\,ds_2 + \frac{1}{2}\langle G\mathbf{E}[x(t_1)],\mathbf{E}[x(t_1)]\rangle.$$

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Here t is the time, t_0, t_1 are given numbers, x is an n-dimensional vector function, A is a real matrix of size $n\times n$, b is an n-dimensional vector, $u(t)$ is a scalar function (control), x_0 is a random vector, $\langle\,\cdot\,,\cdot\,\rangle$ is the inner product, ε is the random process defined by the characteristic functional $\psi(v)$, $B(s_1,s_2)$ is a given self-adjoint nonnegative matrix function of size $n\times n$, $C(s_1,s_2)$ is a given positive function, G is a nonnegative matrix of size $n\times n$, and $\mathbf{E}[x(t)]$ is the expectation of the random process. Consider the function $\chi(s)=\chi(t_0,t,s)$ of s, which is defined by $\operatorname{sign}(s-t_0)$ if s lies in the closed interval with endpoints $\min[t_0,t]$, $\max[t_0,t]$, and is zero otherwise.

THEOREM. *If the expectation $\mathbf{E}[u(t)]$ is a solution of the above problem, then it satisfies the integral Fredholm equation $\int_{t_0}^{t_1} W(s,t)\mathbf{E}[u(s)]\,ds=F(t)$, where*

$$W(s,t)=\int_s^{t_1}\int_t^{t_1}\langle B(s_1,t)\psi(-i\chi(s,s_1)A)\mathbf{E}[x_0],\,\psi(-i\chi(t,s_2)A)b\rangle\,ds_2\,ds_1 + C(s,t)+\langle G\psi(-i\chi(s,t_1)A)b,\,\psi(-i\chi(t,t_1)A)b\rangle,$$

$$F(t)=-\int_{t_0}^{t_1}\int_t^{t_1}\langle B(s_1,s_2)\psi(-i\chi(t_0,s_1)A)\mathbf{E}[x_0],\,\psi(-i\chi(t,s_2)A)b\rangle\,ds_1\,ds_2.$$

A. A. Zamyatin (Lomonosov Moscow State University, Moscow, Russia)**, V. A. Malyshev** (The author is deceased. Former address: Lomonosov Moscow State University, Moscow, Russia)**. Random regular flows of classical particles.**

We consider a system of N interacting particles (of unit mass) $0=x_1(0)<x_2(0)<\cdots<x_N(0)<x_{N+1}(0)=L$ on the circle ($x_1(0)=x_{N+1}(0)$) with interaction potential energy $U=(\omega^2/2)\sum(x_{k+1}-x_k-L/N)^2$, where L is the circle length, and x_k are the particle coordinates. Each particle interacts with neighboring particles and is subject to the dissipative force $-\alpha\dot{x}_k$, $\alpha>0$, and a random driving force $f(t)$, which is a second-order stationary random process ($\mathbf{E}f^2(s)<\infty$) with continuous covariance function and expectation $\overline{f}=\mathbf{E}f(s)$.

It is assumed that the particles exchange velocities at collisions. Hence the original order of particles is preserved at any time.

Let $\mu(du)$ be an orthogonal measure for the centered process $f(s)-\overline{f}$.

THEOREM. (1) *The differences $q_k(t)=x_{k+1}(t)-x_k(t)$ are deterministic variables, and $q_k(t)\to L/N$ as $t\to\infty$.*

(2) *There exists a stationary process*

$$\xi(t)=\frac{\overline{f}}{\alpha}+\int_{\mathbf{R}} e^{itu}(\alpha+iu)^{-1}\,\mu(du)$$

such that, for any $k=1,\dots,N$, with probability 1, $\dot{x}_k(t)-\xi(t)\to 0$ as $t\to\infty$. In particular, if $f(t)\equiv\overline{f}$, then $\dot{x}_k(t)\to\overline{f}/\alpha$.

The rest of the talk is concerned with regularity conditions [1] of the process $\{x_k(t)\}$; the Euler equations are obtained for any time t in the limit $N\to\infty$.

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A. I. Zhdanok (Institute for Information Transmission Problems of the Russian Academy of Sciences (Kharkevich Institute), Moscow, Russia)**, A. K. Khuruma** (Tyva State University, Kyzyl, Russia)**. Decomposition of finitely additive Markov chains and asymptotics of their components.** [16]

We consider Markov chains generated by a finitely additive transition probability $P(x,E)$ defined on an arbitrary discrete space (X,Σ_d). Such Markov chains are studied using the operator approach, where $P(x,E)$ generates a Markov operator A acting in the space of finitely additive measures [1]. We decompose $P(x,E)$ and A in a sum of countably additive and purely finitely additive components: $P = P_{\text{ca}} + P_{\text{pfa}}$ and $A = A_{\text{ca}} + A_{\text{pfa}}$. The following theorem is proved.

Theorem. *Assume that, for a finitely additive combined Markov chain, the set* $Q_y = \{x \in X : P_{\text{ca}}(x,\{y\}) > 0\}$ *is finite for any* $y \in X$. *Then the component* A_{ca} *sends all purely finitely additive measures into the same measures. Moreover, the sequences of norms of countably additive components* $\|\mu_{\text{ca}}^{n+1}\|$ *and purely finitely additive components* $\|\mu_{\text{pfa}}^{n+1}\|$ *for the sequence of measures* $\mu^{n+1} = A\mu^n$ *converge exponentially fast and uniformly to* 0 *and* 1, *respectively.*

The proof of this result, together with some related theorems, is given in [2].

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M. V. Zhitlukhin (Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia)**. Optimal growth strategies in a market model with large number of agents.** [17]

Let strongly continuous positive semimartingales X_t^n, $n = 1,\dots,N$, be defined on a filtered probability space $(\Omega,\mathcal{F},(\mathcal{F}_t)_{t\geqslant 0},\mathbf{P})$.

For continuous adapted processes λ_t, μ_t with values in the set $\Delta^N = \{x \in \mathbf{R}_+^N : x^1 + \dots + x^N = 1\}$ and constants $\rho > 0$, $w_0 > 0$, consider the system of equations

$$V_t = \frac{1}{\rho}\sum_{n=1}^{N} X_t^n, \qquad S_t^n = \mu_t^n V_t \quad (n = 1,\dots,N), \tag{1}$$

$$dW_t = \sum_{n=1}^{N} \frac{\lambda_t^n W_t}{S_t^n}(dS_t^n + X_t^n\,dt) - \rho W_t\,dt, \qquad W_0 = w_0. \tag{2}$$

Theorem. *There exists an a.s. unique process* μ_t *such that, for any process* λ_t, *if* (1), (2) *have a solution* W_t, *then the process* W_t/V_t *is a local martingale.*

This result has the following economic interpretation. Assume that processes X_t^n define the intensities of dividend payments from N assets, and S_t^n define their prices, which are controlled by the strategy μ_t of a representative agent with net asset V_t. Then, for any strategy λ_t of the "small" agent, its asset W_t cannot grow faster than that of the representative agent in the sense that W_t/V_t is a local martingale.

We also show how to find the process μ_t in an explicit form.

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A. V. Zorine (Lobachevsky State University of Nizhny Novgorod, Nizhny Novgorod, Russia)**. On stationary distribution of a service process of nonordinary flows with time separation for algorithm threshold switching.**

We consider the service system from [1]. Assume that a random external environment has one state, the number of input flows m is 2, the input flow Π_j is a Poisson flow of groups, and $\{f(b,j);\, b = 1,2,\dots\}$ is the distribution of probabilities of the group size over the flow Π_j, $j = 1,2$. Let us study conditions for existence and a method for evaluation of the stationary distributions of the homogeneous Markov random process $\{(\Gamma(t), \kappa_1(t), \kappa_2(t));\, t \geqslant 0\}$. Assume that, in the stationary regime,

$$\Psi(z_1, z_2, r) = \mathbf{E}\bigl(z_1^{\kappa_1(t)} z_2^{\kappa_2(t)} I(\Gamma(t) = \Gamma^{(r)})\bigr), \qquad f_j(j) = \sum_{b=1}^{\infty} z_j^b f(b,j),$$

$|z_1| < 1$, $|z_2| < 1$, and $Q(0,0,0) = \mathbf{P}(\{\Gamma(t) = \Gamma^{(0)},\, \kappa_1(t) = 0,\, \kappa_2(t) = 0\})$.

Theorem 1. *For* $r = 1,2$,

$$\begin{aligned}
&\Psi(z_1, z_2, r)\bigl(\lambda_1(f_1(z_1) - 1) + \lambda_2(f_2(z_2) - 1) - \beta_r\bigr)\\
&\qquad + \sum_{j=1}^{2} \overline{\beta}_j \mathbf{E}\bigl(z_1^{\kappa_1(t)} z_2^{\kappa_2(t)} I\bigl(\{\Gamma(t) = \Gamma^{(2+j)},\, h(\kappa_1(t), \kappa_2(t)) = r\}\bigr)\bigr)\\
&\qquad + \lambda_r f_r(z_r) Q(0,0,0) = 0,\\
&\Psi(z_1, z_2, 2+r)\bigl(\lambda_1(f_1(z_1) - 1) + \lambda_2(f_2(z_2) - 1) - \overline{\beta}_r\bigr)\\
&\qquad + \beta_r z_r^{-1}\bigl(1 + p_{r,1}(z_1 - 1) + p_{r,2}(z_2 - 1)\bigr)\Psi(z_1, z_2, r) = 0,\\
&\qquad\qquad (\lambda_1 + \lambda_2) Q(0,0,0) = \overline{\beta}_1 Q(3,0,0) + \overline{\beta}_2 Q(4,0,0).
\end{aligned}$$

The equations in Theorem 1 are solved for the threshold switch function: $h(x_1, x_2)$ is 1 for $x_1 > L \in \{0,1,\dots\}$ or for $1 \leqslant x_1 \leqslant L$ and $x_2 = 0$; 2 for $x_1 \leqslant L$ and $x_2 \geqslant 1$; and 0 for $x_1 = x_2 = 0$.

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